Final Report

covering the time November 1, 1958 to October 31, 1962

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Contract Title

INVESTIGATION OF OPTIMUM CONTACTOR CONTROL

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Investigation of Optimum

Contactor Control

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The investigation started in November 1958 with a budget for one year, a continuation for three more years was granted in 1959.

Purpose of the investigation

The determination of switching functions which allow to control a linear system of higher than second order in an optimum fashion. The system should have complex and real poles, or complex poles only. The performance criterion was not specified, but the choice was left to the investigator.

The switching function should be expressed as function of the phase or state variables, not as a function of time. This form allows the design of a programmed feedback control.

Results of the investigation

On the basis of Bushaw's investigation of time-optimal control of second-order systems with complex poles and the experiences of the principal investigator with systems of second, third and fourth-order with complex poles and linear switching functions, trajectories of third-order systems were studied in the phase space.

Pontryagin's maximum principle became known to the investigator at the beginning of 1959. This principle gives a systematic procedure for finding the control function u as function of time for a given plant and a given performance criterion. It shows that the minimum settling-time control of linear systems is a contactor control.

This principle gave guidance, but it contained one essential mathematical difficulty. For finding the control function $u(t)$ one needs to know the initial conditions for the solutions of the system which is adjoint to the system to be controlled. There is no simple explicit relation between the initial disturbances of the system and the initial conditions of the adjoints. In addition there is still the task to find u as function of the phase or state variables.

In the second-order minimum settling-time case Bushaw found the locus for all switching points in the state space. This knowledge frees the control designer of the determination of the initial values of the adjoints. Following this idea the investigator and his students started a systematic search for the locus of all switching points in the state space for third-order system, when time-optimal control was applied. It was found that the switching surface for systems with the transfer function $1/s(s^2+1)$ could be described easily by curves lying on it and carrying the switching points for certain given disturbances. A simple iteration procedure for finding the correct switching curve for given initial disturbances was found. For systems $1/(s+\gamma)(s^2+2\zeta s+1)$ the procedure does not give the exact switching points in the state space but a good approximation. Phase-point trajectories obtained with exact (optimal) and with approximate (sub-optimal) controls were compared, and the deviations were small for $\gamma = 0$ and could be kept reasonably small for $\gamma \neq 0$.

However, the investigation showed clearly that realization of such a control was rather complicated. Only high speed digital computing means would allow to build such a control system.

While studying so many examples it became clear that the switching surface of a third-order system with two complex poles had a complicated structure which could be decomposed in a main structure and an added fine structure. It became obvious that this fine structure is important only near the origin of the state space. This idea was systematically exploited. It was found that the "main" structure "for the $1/s(s^2+1)$ system could be described by a ruled surface (this is a surface generated by straight lines). A digital control was then developed, which employs two switching surfaces. The "outer" surface, corresponding to the main structure, is used in that region of the state space, where the norm of the state vector is larger than a given number, and the "inner" surface is used near the state-space origin.

In addition to these studies, the short time assistance of a young mathematician led to an interesting interpretation of Pontryagin's maximum principle as an efflux of Huyghens' principle. A report about this idea was written.

During the end of the contract time the performance criterion

$$J = \int_0^T |u| dt \rightarrow \text{minimum}$$

gained importance. If the attitude of satellites is controlled with the help of jets, J measures the fuel consumption. T denotes the chosen time for reducing a disturbance to zero ($T > T^*$ = minimum settling time) .

Many papers have been written on this subject. It is known that the jets should be blowing at full force in one direction or the opposite direction, but "coasting" (no use of the jets) is also required. That means $u(t)$ can assume values $[+1, 0, -1]$, or vice versa or any subgroup of this.

We found that for a second-order system, there are certain times T_v which allow a particularly simple arrangement of the switching points in the phase plane. Since these T_v are spaced at regular intervals, their choice does not impose a serious restriction. The realization of such a control with analog computer elements is easy. Experiments were successful. A paper will be written on this subject.

This last investigation was done for a second-order system for gaining experience for more general cases.

Publications

1) I. Flügge-Lotz and Mih Yin, "On the Optimum Response of Third-Order Contactor Control Systems," Stanford University Engineering Mechanics Techn. Report 125, April 1960. An abbreviated version was published in Journal of Basic Eng., ASME Trans., 83, D, 1961, pp. 59-64.

2) I. Flugge-Lotz and H. A. Titus, Jr., "The Optimum Response of Full Third-Order Systems with Contactor Control," J. of Basic Eng., ASME Trans., 84, D, Dec. 1962, pp. 554-558.

3) I. Flügge-Lotz, "Synthesis of Third-Order Contactor Control Systems," Proceedings of the First International Federation of Automatic Control Congress 1960, Moscow, U.S.S.R..

4) I. Flügge-Lotz and H. A. Titus, Jr., "Optimum and Quasi-Optimum Control of Third and Fourth-Order Systems," paper accepted for International Federation of Automatic Control Congress 1963, Basle, Switzerland.

5) I. Flügge-Lotz and H. Halkin, "Pontryagin's Maximum Principle and Optimal Control, Engineering Mechanics Techn. Report 130, Stanford University (AFOSR TN 1489) 1961.

6) I. Flügge-Lotz and Andrew Craig, "The Choice of Time for Zeroing a Disturbance in Minimum Fuel-Consumption Control Problems," to be published.

Attended Meetings

The principal investigator received support from this contract to attend the following meetings:

1) JACC 1960, Massachusetts Institute of Technology.

2) International Federation of Automatic Control Congress in Moscow June 27-July 7, 1960 (partial support; additional support from NASA).

3) Meeting: "Contractor's Critique of First IFAC Congress," Baltimore, August 1960.

4) OSR-RIAS Symposium on "Nonlinear Differential Equations and Nonlinear Mechanics," Air Force Academy, Colorado Springs, August 1961.

5) Optimum Systems Synthesis Conference, Wright-Patterson Air Force Base, Ohio, September 1962.

Additional Information

Two graduate students received support for their research work towards the Ph.D. degree. In addition two other graduate students received short-time support (during summer quarter).

I. Flügge-Lotz